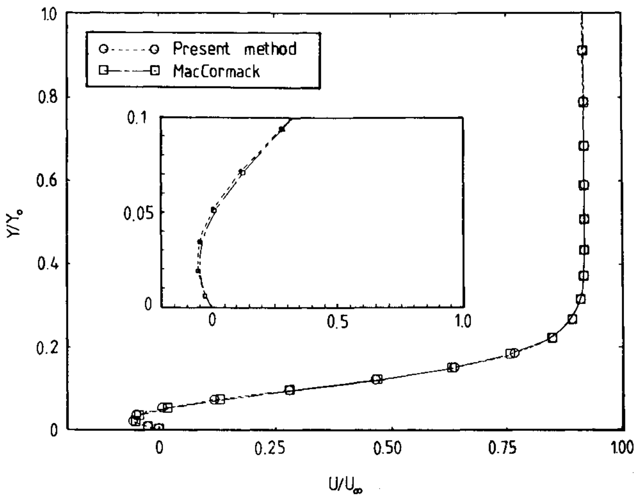

Fig. 2 Velocity profile in the separation region. The largest separation is shown; U_∞ is the velocity at the inflow boundary.

Two-Dimensional Flow

Although the SIMPLE algorithm has been described within the framework of a one-dimensional coordinate system, the basic method, together with the nonstandard features discussed in the previous section, can easily be extended to two or three dimensions. The equations of motion for two-dimensional viscous compressible flow and the extension of the original SIMPLE method to two dimensions are discussed by Van Doornmaal et al.[6] Because of the lack of space, the extension of the one-dimensional scheme to two dimensions will not be discussed in detail here. The results of a shock-boundary interaction are, however, given.

The geometry and flowfield representing the interaction are discussed by MacCormack.[1] An externally generated shock wave is incident upon the boundary layer of a flat plate. For a strong enough shock wave, the boundary layer will separate from the surface of the plate and reattach downstream. A region of rotating fluid exists between the separation and reattachment points that causes the boundary layer to thicken and generate a series of compression and expansion waves that eventually form the reflected shock wave. According to MacCormack[1] the separation region is fairly sensitive to calculation and therefore serves as a good test for a numerical method.

In Fig. 2 the calculated velocity distribution in the separation region with the present method, using a nonstaggered grid, is compared with the calculated results of the 1969 explicit method of MacCormack[1] for a shock angle of 32.585 deg and a Reynolds number of 2.9×10^5. The agreement between the two methods is good.

Concluding Remarks

For one-dimensional flow through a nozzle, it can be deduced that a mixed-differencing scheme for the convection terms and a central-differencing scheme for densities in the continuity equation considerably improve the accuracy of the SIMPLE method. For the shock-boundary interaction problem, the SIMPLE method for a nonstaggered grid with mixed differencing for the convection terms and central differencing for densities compares very well with the method of MacCormack.

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Calculation of Matched Pressure Properties of Low-Altitude Rocket Plumes

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Nomenclature

A = area, m
D = plume drag, N
F = actual rocket thrust, N
F_{max} = maximum possible rocket thrust, N
$\dot{m}$ = rocket exhaust mass flow, kg/s
p = pressure, Pa
R = gas constant, J/kg/K
r = plume boundary radius, m
s = distance along plume boundary, m
T = temperature, K
u = axial velocity, m/s
γ = ratio of specific heats
θ = angle of plume boundary relative to plume axis, rad

Subscripts

b = station at which the pressure within the plume is uniform and equal to the ambient pressure
c = rocket combustion chamber
e = nozzle exit plane
∞ = limiting state where p and T are absolute 0

Introduction

An accurate estimate of the temperature of the exhaust gas in the plume of a rocket at low altitude is required to predict the plume's infrared emission. Sukanek[1] proposed an approximate one-dimensional method of calculating the temperature based on the Jarvinen and Hill universal plume model. Pearce and Dash[2] corrected an inconsistency in Sukanek's approach but introduced two of their own. This Note draws attention to these errors and demonstrates that the universal plume model is inappropriate for this problem.

Discussion

Consider a control volume (cv) drawn along the boundary between the airstream and the exhaust plume of a rocket, Fig. 1. The exhaust gas enters the cv parallel with the nozzle axis and at uniform pressure p_e (the nozzle exit pressure). The gas leaves the cv far downstream in an axial direction at the pressure of the surrounding atmosphere p_b. Mixing with the airstream and acceleration of the rocket are neglected and the momentum balance is simply

$$\int_0^{S_b} 2\pi r \sin\theta (p - p_b)\, ds = \dot{m}(u_b - u_e) + (p_b - p_e)A_e = \dot{m}u_b - F \quad (1)$$

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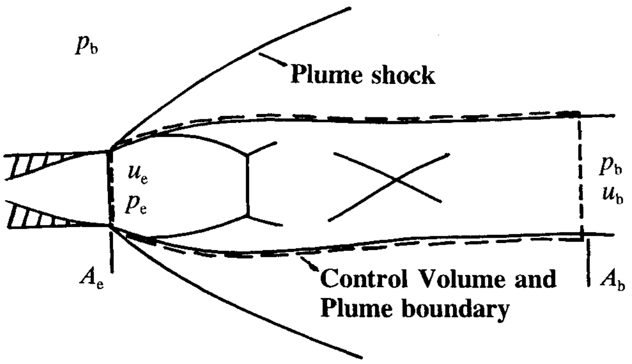

Fig. 1 Control volume analysis of a low-altitude plume.

The integral in Eq. (1) is the axial force applied by the freestream and is termed the plume drag, D. Pearce and Dash, following Sukanek, estimated its magnitude from Jarvinen and Hill's universal plume model[2] for which

$$\frac{D}{F} = \left(\frac{C_{F_{\max}}}{C_F}\right) - 1 = \frac{F_{\max} - F}{F} \tag{2}$$

For this model it can be seen that D is independent of the rocket's speed and largely determined by the exit Mach number of the rocket nozzle. The model has been applied successfully to the estimation of plume size and shape but the drag dependence is weak (quarter power) for that problem.[3] Its application to the matched pressure problem is examined subsequently.

The maximum thrust $F_{\max}$ that a rocket engine can produce is equal to the maximum momentum that it can impart to the exhaust gas, and thus the maximum thrust coefficient is normally defined as

$$C_{F_{\max}} = \dot{m} u_\infty / p_c A^* \tag{3}$$

For a gas with a constant ratio of specific heats this can be written

$$C_{F_{\max}} = \left\{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/(\gamma-1)} \right\}^{1/2} \tag{4}$$

This is the definition of $C_{F_{\max}}$ used in the universal plume model.[1] However, in their application of the model, Pearce and Dash redefined $C_{F_{\max}}$ as

$$C_{F_{\max}} = \left\{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/(\gamma-1)} \left[1 - \left(\frac{p_b}{p_c} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2} \tag{5}$$

The extra factor is equivalent to u/u_∞, where u is the velocity obtained in an isentropic expansion to p_b (with constant γ). Therefore, Pearce and Dash have not used the universal plume model but have effectively assumed that the plume drag is just sufficient to allow an isentropic expansion of the exhaust gas to the ambient pressure. This draws attention to the second inconsistency in their paper, because in their Fig. 1, Pearce and Dash show a difference between plume temperature calculated with the "corrected theory" and that calculated assuming an isentropic expansion. The two approaches are identical and the difference in result arises from the assumption of constant γ in the corrected theory and the use of a set of nozzle exit and stagnation conditions which are not consistent with constant γ. The plume flowfield and rocket thrust are of course determined by the nozzle exit conditions regardless of what processes took place in the expansion up to that point. However, if one refers to the conditions in the combustion chamber when calculating the rocket thrust then the nature of the expansion in the nozzle must be taken into account. Pearce and Dash failed to do so and produced a spurious result for the plume temperature which quite coincidentally agreed with their numerical calculation.

Conclusion

When the universal plume model is used correctly to estimate the plume drag so that the temperature in the matched pressure plume can be calculated, the rather uninteresting result is $T_b = 0$ K. The example application presented by Pearce and Dash (their Fig. 1) shows that the self-consistent simple model of an isentropic expansion from p_e to p_b gave results not too far removed from the numerical calculations. It may be possible to improve the isentropic model slightly by adding some fraction of the total possible isenthalpic entropy increase, $R \ln (p_c/p_b)$. However, given the sensitivity of the infrared emission to the plume temperature, it may be that only a full numerical simulation will give sufficiently accurate results.

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Analysis of Interlaminar Stresses in the Torsion of Symmetric Laminates

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Introduction

In a recent paper[1] a modified shear deformation theory (SDT) based on Reissner-type assumptions[2] was considered in obtaining an approximate solution for the torsion of a symmetrically laminated, anisotropic plate. In-plane stresses were modified to satisfy free-edge boundary conditions. Interlaminar stresses were obtained by integrating the equations of equilibrium. Thus, the resulting stresses satisfied point-by-point equilibrium as well as the correct boundary conditions.

This same procedure has also been applied to the case of a [0/90 deg]$_s$ laminate under uniaxial tension loading.[3] However, numerical results indicate that the maximum value of the interlaminar normal stress, σ_3, at the the laminate midplane underestimates the value obtained from theory of elasticity. In addition, the distance away from the free edge over which the interlaminar normal stress dissipates was greater than that obtained from the elasticity solution.

These discrepancies are attributed to the vanishing of the interlaminar normal strain component associated with the modified shear deformation theory. This conclusion is supported by the work of Pagano.[4] In particular, he obtained excellent results for the distribution of σ_3 at the midplane of a [0/90 deg]$_s$ laminate by utilizing a shear deformation theory in conjunction with a thickness-stretch mode (linear transverse displacement through the thickness). His procedure involved treating the upper half of the composite as a laminated plate with the distribution of σ_3 at the midplane being an unknown function. Although this approach produced excellent results for the distribution of σ_3 at the midplane, it did not provide for an accurate calculation of σ_3 through

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